Torsion nonminimally coupled to the electromagnetic field and birefringence

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Abstract

In conventional Maxwell–Lorentz electrodynamics, the propagation of light is influenced by the metric, not, however, by the possible presence of a torsion T. Still the light can feel torsion if the latter is coupled nonminimally to the electromagnetic field F by means of a supplementary Lagrangian of the type $\sim \ell^2 T^2 F^2$ (ℓ = coupling constant). Recently Preuss suggested a specific nonminimal term of this nature. We evaluate the spacetime relation of Preuss in the background of a general O(3)-symmetric torsion field and prove by specifying the optical metric of spacetime that this can yield birefringence in vacuum. Moreover, we show that the nonminimally coupled homogeneous and isotropic torsion field in a Friedmann cosmos affects the speed of light.

Keywords: Torsion, light propagation, nonminimal coupling, birefringence, speed of light

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I. "ADMISSIBLE" COUPLINGS OF TORSION

Maxwell's equations $dH = J$ and $dF = 0$, with the excitation $H = (\mathcal{H}, \mathcal{D})$ and the field strength $F = (E, B)$, are generally covariant. If supplemented by the Maxwell–Lorentz spacetime relation $H = \lambda_0 {}^\star F$ (the star denotes the Hodge operator and λ_0 the vacuum impedance), one arrives at the standard Maxwell–Lorentz vacuum electrodynamics and can study, for example, the propagation of electromagentic disturbances ("light") in vacuum. Clearly, the metric is then the only geometrical quantity light couples to. In particular, torsion T^α and nonmetricity $Q_{\alpha\beta}$ are completely "transparent" to light, as has been discussed, for example, by Puntigam et al. [1]. Light doesn't interact with these fields under the assumptions specified.

Torsion can affect light when either the Maxwell equations $dH = J$ and $dF = 0$ (at least one of them) or the Maxwell–Lorentz spacetime relation $H = \lambda_0 {}^\star F$ are modified. A modification of the inhomogeneous Maxwell equation could read $[d + (e_\alpha \rfloor T^\alpha)]H = J$, with the frame e_α and the interior product sign $\rfloor$; see in this context [2,3]. This *nonminimal* coupling of torsion to the Maxwell equations we call "inadmissible" [4] since it violates the well-established conservation law of electric charge. The same can be said, *mutatis mutandis*, with respect to the homogeneous equation $dF = 0$ and the conservation law of magnetic flux.

Therefore, we turn to the "admissible" nonminimal couplings that modify the Maxwell–Lorentz spacetime relation. One possible example is, see [4]:

$$H = \lambda_0 \left[1 + \gamma^2 \, {}^\star(T^\alpha \wedge T_\alpha)\right] {}^\star F \,. \tag{1}$$

Such a dilaton type coupling preserves the light cone structure. Other quadratic torsion coupling are also possible, see [5] for a complete list. It should be understood that the Maxwell equations $dH = J$ and $dF = 0$ are left intact, with electric charge and magnetic flux still conserved, but the spacetime relation (1) with a new constant of nature γ would induce a coupling of light to torsion; here $[\gamma] = \textit{length}$. Note that the spacetime relation (1)

is still linear in F but a quadratic torsion term emerges; also quartic torsion terms would be possible.

II. PREUSS' COUPLING TO A BACKGROUND TORSION

Recently Preuss in his thesis [6] and in a talk [7] suggested a different non-minimal coupling term. His supplementary Lagrangian 4-form

$$L_{\text{Preuss}} = \lambda_0 \ell^2\,{}^\star(T_\alpha \wedge F)\; T^\alpha \wedge F \tag{2}$$

(cf. Eq.(1.59) of [6]) corresponds to the spacetime relation

$$H = \lambda_0 \left[{}^\star F - 2\ell^2\,{}^\star(T_\alpha \wedge F)\; T^\alpha\right], \tag{3}$$

since $L = -\frac{1}{2}\,H \wedge F$. The coupling constant ℓ has the dimension of length. Eq. (3) should be compared with (1). Both spacetime relations are options but for parity reasons (3) looks better. Alternatively, one could also modify (1) by inserting a star within the parenthesis: $\gamma^2\,{}^\star(T^\alpha \wedge T_\alpha) \longrightarrow \gamma^2\,{}^\star(T^\alpha \wedge {}^\star T_\alpha)$. We will not discuss the most general form of such quadratic torsion couplings here, see, however, Itin [5]. For concreteness, we confine our attention to the Preuss Lagrangian. Hence from now on we assume the spacetime relation (3).

Preuss made his ansatz since he was on a search for new astronomical tests of Einstein's equivalence principle. In addition, he took an exact solution found by Tresguerres [8,9] within the metric-affine theory of gravity (MAG); for MAG see the review [10]. The main ingredient for these considerations is a O(3)-symmetric torsion that is given by ($\vartheta^{\hat0\hat1} := \vartheta^{\hat0}\wedge\vartheta^{\hat1}$ etc., with ϑ^α as coframe)

$$T^\alpha\big|_{\mathrm{O}(3)} = \begin{pmatrix} T_{\hat0\hat1}{}^{\hat0}\,\vartheta^{\hat0\hat1} \\ T_{\hat0\hat1}{}^{\hat1}\,\vartheta^{\hat0\hat1} \\ T_{\hat0\hat2}{}^{\hat2}\,\vartheta^{\hat0\hat2} + T_{\hat1\hat2}{}^{\hat2}\,\vartheta^{\hat1\hat2} \\ T_{\hat0\hat3}{}^{\hat3}\,\vartheta^{\hat0\hat3} + T_{\hat3\hat1}{}^{\hat3}\,\vartheta^{\hat3\hat1} \end{pmatrix} = \frac{1}{\ell}\begin{pmatrix} f\,\vartheta^{\hat0\hat1} \\ -h\,\vartheta^{\hat0\hat1} \\ -k\,\vartheta^{\hat0\hat2} + g\,\vartheta^{\hat1\hat2} \\ -k\,\vartheta^{\hat0\hat3} - g\,\vartheta^{\hat3\hat1} \end{pmatrix}. \tag{4}$$

We introduced an overall ℓ^{-1} factor in (4) in order to keep the functions f, g, h, k dimensionless. For the torsion with its 24 components only 4 functions are left open in the case of O(3)-symmetry. Preuss took $f(r), g(r), h(r), k(r)$ from [8,9] and computed the corresponding birefrigence by using a formalism of Haugan and Kauffmann [11] (see also Gabriel et al. [12]).

III. BIREFRINGENCE FOR AN ARBITRARY SPHERICALLY SYMMETRIC TORSION

We have shown in the past (see [13,14]) how one can determine light propagation in an arbitrary spacetime by means of a generalized Fresnel equation provided a *linear* spacetime relation $H = \kappa(F)$, or in components $H_{\alpha\beta} = \frac{1}{2}\,\kappa_{\alpha\beta}{}^{\gamma\delta}\,F_{\gamma\delta}$, is specified. Recently, we applied this method also to nonlinear electrodynamics, see [15]. Here we demonstrate our procedure for the spherically symmetric torsion case, but an extension to axial symmetry is straightforward. In this way we confirm Preuss' results in a more general framework and in a very direct manner.

A. Modified constitutive tensor density of spacetime

We start by putting (3) into component form. Field strength and torsion decompose as $F = F_{\alpha\beta}\,\vartheta^{\alpha\beta}/2$ and $T^\alpha = T_{\beta\gamma}{}^\alpha\,\vartheta^{\beta\gamma}/2$, respectively. Then (3) reads

$$H = \frac{1}{2}\,H_{\alpha\beta}\,\vartheta^{\alpha\beta} = \frac{\lambda_0}{2}\left[F_{\alpha\beta}\,{}^\star\vartheta^{\alpha\beta} - \ell^2\,{}^\star(T_\alpha \wedge \vartheta^{\beta\gamma})\,T_{\mu\nu}{}^\alpha\,F_{\beta\gamma}\,\vartheta^{\mu\nu}\right]. \tag{5}$$

Because of ${}^\star\vartheta^{\alpha\beta} = \eta^{\alpha\beta}{}_{\gamma\delta}\,\vartheta^{\gamma\delta}/2$, here $\eta^{\alpha\beta\gamma\delta}$ is the totally antisymmetric unit tensor, we find

$$\kappa_{\gamma\delta}{}^{\alpha\beta} = \lambda_0\left[\eta^{\alpha\beta}{}_{\gamma\delta} - 2\ell^2\,{}^\star(T_\mu \wedge \vartheta^{\alpha\beta})\,T_{\gamma\delta}{}^\mu\right]. \tag{6}$$

Moreover, we decompose the torsion 2-form T_μ and introduce the components of the Hodge dual of the torsion ${}^\star T^\alpha = \check{T}_{\beta\gamma}{}^\alpha\,\vartheta^{\beta\gamma}/2$. Then the constitutive tensor of spacetime becomes

$$\kappa_{\gamma\delta}{}^{\alpha\beta} = \lambda_0\left(\eta^{\alpha\beta}{}_{\gamma\delta} - 2\ell^2\,\check{T}^{\alpha\beta}{}_\mu\,T_{\gamma\delta}{}^\mu\right). \tag{7}$$

We can raise the two first indices with the η according to the definition (see [13])

$$\chi^{\gamma\delta\alpha\beta} = -\frac{\sqrt{-g}}{2}\,\eta^{\gamma\delta\lambda\nu}\,\kappa_{\lambda\nu}{}^{\alpha\beta}\,. \tag{8}$$

Then, after some algebra, we find

$$\chi^{\alpha\beta\gamma\delta} = 2\lambda_0\,\sqrt{-g}\,\Big(g^{\alpha[\gamma}\,g^{\delta]\beta} + \ell^2\,\check{T}^{\alpha\beta}{}_{\mu}\,\check{T}^{\gamma\delta\mu}\Big)\,. \tag{9}$$

Because of the symmetries

$$\chi^{\alpha\beta\gamma\delta} = \chi^{\gamma\delta\alpha\beta} = -\chi^{\beta\alpha\gamma\delta} = -\chi^{\alpha\beta\delta\gamma}\,, \tag{10}$$

the tensor density χ has 21 independent components. Its totally antisymmetric (axion) piece

$$\chi^{[\alpha\beta\gamma\delta]} = \frac{\lambda_0\ell^2}{6}\,\sqrt{-g}\,\eta^{\alpha\beta\gamma\delta}\,\check{T}^{\mu\nu\rho}\,T_{\mu\nu\rho}\,, \tag{11}$$

carries 1 independent component. It is known (see [13]) that the axion does *not* affect a path of light (although the polarization vector feels the axion, see [3]). Hence the principal piece $\chi^{\alpha\beta\gamma\delta} - \chi^{[\alpha\beta\gamma\delta]}$ alone with its 20 independent components determines the light cone structure.

B. Derivation of the double light cone structure

After χ is known explicitly, see (9), we determine the corresponding TR-tensor density[1] $\mathcal{G}$. In holonomic coordinates (natural frames), we have

$$\mathcal{G}^{ijkl} := \frac{1}{4!}\,\hat{\epsilon}_{mnpq}\,\hat{\epsilon}_{rstu}\,\chi^{mnr(i}\,\chi^{j|ps|k}\,\chi^{l)qtu}, \qquad \mathcal{G}^{ijkl} = \mathcal{G}^{(ijkl)} \tag{12}$$

(see [13] and [14], for example; in the framework of general relativity, Perlick [16] describes related methods).

For the O(3)-symmetric torsion of (4), we now choose an orthonormal frame with the metric $g_{\alpha\beta} = \mathrm{diag}(+1,-1,-1,-1)$. Application of the Hodge star to (4) yields

[1] The name Tamm-Rubilar tensor density was suggested in [13], see the references given there.

$$
{}^\star T^\alpha|_{\mathrm{O}(3)} = \begin{pmatrix} T_{\hat 0\hat 1}{}^{\hat 0}\,{}^\star\vartheta^{\hat 0\hat 1} \\ T_{\hat 0\hat 1}{}^{\hat 1}\,{}^\star\vartheta^{\hat 0\hat 1} \\ T_{\hat 0\hat 2}{}^{\hat 2}\,{}^\star\vartheta^{\hat 0\hat 2} + T_{\hat 1\hat 2}{}^{\hat 2}\,{}^\star\vartheta^{\hat 1\hat 2} \\ T_{\hat 0\hat 3}{}^{\hat 3}\,{}^\star\vartheta^{\hat 0\hat 3} + T_{\hat 3\hat 1}{}^{\hat 3}\,{}^\star\vartheta^{\hat 3\hat 1} \end{pmatrix} = \frac{1}{\ell}\begin{pmatrix} -f\,\vartheta^{\hat 2\hat 3} \\ h\,\vartheta^{\hat 2\hat 3} \\ k\,\vartheta^{\hat 3\hat 1} + g\,\vartheta^{\hat 0\hat 3} \\ k\,\vartheta^{\hat 1\hat 2} - g\,\vartheta^{\hat 0\hat 2} \end{pmatrix} \tag{13}
$$

(recall that $\eta^{\hat 0\hat 1\hat 2\hat 3} = -1/\sqrt{-g}$). We read off the following components:

$$
\begin{aligned}
&\check T_{\hat 2\hat 3}{}^{\hat 0} = -f/\ell\,, \quad \check T_{\hat 2\hat 3}{}^{\hat 1} = h/\ell\,, \quad \check T_{\hat 3\hat 1}{}^{\hat 2} = k/\ell\,, \\
&\check T_{\hat 0\hat 3}{}^{\hat 2} = g/\ell\,, \quad \check T_{\hat 1\hat 2}{}^{\hat 3} = k/\ell\,, \quad \check T_{\hat 0\hat 2}{}^{\hat 3} = -g/\ell\,.
\end{aligned} \tag{14}
$$

We substitute (14) into (9):

$$
\begin{aligned}
&\chi^{\hat 0\hat 1\hat 0\hat 1} = -\lambda_0\,, \quad \chi^{\hat 0\hat 2\hat 0\hat 2} = \chi^{\hat 0\hat 3\hat 0\hat 3} = -\lambda_0(1+2g^2)\,, \\
&\chi^{\hat 2\hat 3\hat 2\hat 3} = \lambda_0[1+2(f^2-h^2)]\,, \quad \chi^{\hat 3\hat 1\hat 3\hat 1} = \chi^{\hat 1\hat 2\hat 1\hat 2} = \lambda_0(1-2k^2)\,, \\
&\chi^{\hat 0\hat 2\hat 1\hat 2} = \chi^{\hat 1\hat 2\hat 0\hat 2} = -\chi^{\hat 0\hat 3\hat 3\hat 1} = -\chi^{\hat 3\hat 1\hat 0\hat 3} = -2\lambda_0 g\,k\,.
\end{aligned} \tag{15}
$$

Obviously, the axion part (11) vanishes for the spherically symmetric torsion: $\chi^{[\alpha\beta\gamma\delta]} = 0$. In 6×6 form (the corresponding matrix is symmetric), we have (with the bivector indices $I, J = 1,\dots,6$ defined as $1 = \hat 0\hat 1, 2 = \hat 0\hat 2, 3 = \hat 0\hat 3, 4 = \hat 2\hat 3, 5 = \hat 3\hat 1, 6 = \hat 1\hat 2$)

$$
\chi^{IK} = \begin{pmatrix} B & D \\ C & A \end{pmatrix}, \tag{16}
$$

where the 3×3 blocks read:

$$
A = -\,\lambda_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1+2g^2 & 0 \\ 0 & 0 & 1+2g^2 \end{pmatrix}, \quad C = \lambda_0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -2gk \\ 0 & 2gk & 0 \end{pmatrix}, \tag{17}
$$

$$
B = \;\lambda_0 \begin{pmatrix} 1+2(f^2-h^2) & 0 & 0 \\ 0 & 1-2k^2 & 0 \\ 0 & 0 & 1-2k^2 \end{pmatrix}, \quad D = C^{\mathrm{T}}. \tag{18}
$$

The corresponding Fresnel equation determines the components of the wave covector (1-form) q_i of the propagating wave:

$$
\mathcal{G}^{ijkl} q_i q_j q_k q_l = 0\,. \tag{19}
$$

Explicitly, it is found in this case to be

$$-\lambda_0^3\Big[(1+2g^2)q_{\hat 0}^2+4gk\,q_{\hat 0}\,q_{\hat 1}-(1-2k^2)q_{\hat 1}^2-[1+2(g^2-k^2)](q_{\hat 2}^2+q_{\hat 3}^2)\Big]\times$$
$$\Big[(1+2g^2)q_{\hat 0}^2+4gk\,q_{\hat 0}\,q_{\hat 1}-(1-2k^2)q_{\hat 1}^2-[1+2(f^2-h^2)](q_{\hat 2}^2+q_{\hat 3}^2)\Big]=0. \qquad (20)$$

This result was also checked with the help of computer algebra. Thus, we verify that in this case we have birefringe, i.e., a double lightcone structure. The respective *optical* metrics are, up to a conformal factor, given by

$$g_{(1)}^{\alpha\beta}=\begin{pmatrix} 1+2g^2 & 2gk & 0 & 0\\ 2gk & -1+2k^2 & 0 & 0\\ 0 & 0 & -1+2(k^2-g^2) & 0\\ 0 & 0 & 0 & -1+2(k^2-g^2)\end{pmatrix}, \qquad (21)$$

$$g_{(2)}^{\alpha\beta}=\begin{pmatrix} 1+2g^2 & 2gk & 0 & 0\\ 2gk & -1+2k^2 & 0 & 0\\ 0 & 0 & -1+2(h^2-f^2) & 0\\ 0 & 0 & 0 & -1+2(h^2-f^2)\end{pmatrix}. \qquad (22)$$

IV. TORSION IN FRIEDMANN COSMOLOGY

As another example, we consider light propagation on the background of torsion in a Friedmann universe. The isotropy group is SO(3). The torsion pieces left over after we impose the homogeneity and isotropy conditions read, with $u=u(t)$ and $v=v(t)$ (see [17], [18], and [19], e.g.):

$$T^\alpha|_{\rm F}=\begin{pmatrix} 0\\ T_{\hat 0\hat 1}{}^{\hat 1}\,\vartheta^{\hat 0\hat 1}+T_{\hat 2\hat 3}{}^{\hat 1}\,\vartheta^{\hat 2\hat 3}\\ T_{\hat 0\hat 2}{}^{\hat 2}\,\vartheta^{\hat 0\hat 2}+T_{\hat 3\hat 1}{}^{\hat 2}\,\vartheta^{\hat 3\hat 1}\\ T_{\hat 0\hat 3}{}^{\hat 3}\,\vartheta^{\hat 0\hat 3}+T_{\hat 1\hat 2}{}^{\hat 3}\,\vartheta^{\hat 1\hat 2}\end{pmatrix}=\frac{1}{\ell}\begin{pmatrix} 0\\ u\,\vartheta^{\hat 0\hat 1}+v\,\vartheta^{\hat 2\hat 3}\\ u\,\vartheta^{\hat 0\hat 2}+v\,\vartheta^{\hat 3\hat 1}\\ u\,\vartheta^{\hat 0\hat 3}+v\,\vartheta^{\hat 1\hat 2}\end{pmatrix}. \qquad (23)$$

By simple algebra we find

$$u^2 = \frac{\ell^2}{9} T_{\alpha\beta}{}^{\beta}\, T^{\alpha\gamma}{}_{\gamma}\,, \qquad v^2 = \frac{\ell^2}{6} T_{[\alpha\beta\gamma]}\, T^{[\alpha\beta\gamma]}\,. \tag{24}$$

The dual of (23),

$$^\star T^\alpha|_{\rm F} = \frac{1}{\ell} \begin{pmatrix} 0 \\ v\,\vartheta^{\hat{0}\hat{1}} - u\,\vartheta^{\hat{2}\hat{3}} \\ v\,\vartheta^{\hat{0}\hat{2}} - u\,\vartheta^{\hat{3}\hat{1}} \\ v\,\vartheta^{\hat{0}\hat{3}} - u\,\vartheta^{\hat{1}\hat{2}} \end{pmatrix}, \tag{25}$$

allows us to read off the components of the dual of the torsion:

$$\check{T}_{\hat{0}\hat{1}}{}^{\hat{1}} = \check{T}_{\hat{0}\hat{2}}{}^{\hat{2}} = \check{T}_{\hat{0}\hat{3}}{}^{\hat{3}} = v(t)/\ell\,, \quad \check{T}_{\hat{2}\hat{3}}{}^{\hat{1}} = \check{T}_{\hat{3}\hat{1}}{}^{\hat{2}} = \check{T}_{\hat{1}\hat{2}}{}^{\hat{3}} = -u(t)/\ell. \tag{26}$$

We substitute this result into (9):

$$\chi^{\hat{0}\hat{1}\hat{0}\hat{1}} = \chi^{\hat{0}\hat{2}\hat{0}\hat{2}} = \chi^{\hat{0}\hat{3}\hat{0}\hat{3}} = -\lambda_0(1+2v^2)\,, \tag{27}$$

$$\chi^{\hat{2}\hat{3}\hat{2}\hat{3}} = \chi^{\hat{3}\hat{1}\hat{3}\hat{1}} = \chi^{\hat{1}\hat{2}\hat{1}\hat{2}} = \lambda_0(1-2u^2)\,, \tag{28}$$

$$\chi^{\hat{0}\hat{1}\hat{2}\hat{3}} = \chi^{\hat{0}\hat{2}\hat{3}\hat{1}} = \chi^{\hat{0}\hat{3}\hat{1}\hat{2}} = \chi^{\hat{2}\hat{3}\hat{0}\hat{1}} = \chi^{\hat{3}\hat{1}\hat{0}\hat{2}} = \chi^{\hat{1}\hat{2}\hat{0}\hat{3}} = -2\lambda_0\, uv\,. \tag{29}$$

In contrast to the spherically symmetric case, here the axion part (11) is nontrivial: $\chi^{[\alpha\beta\gamma\delta]} = \alpha\,\epsilon^{\alpha\beta\gamma\delta}$ with the axion field $\alpha = -2\lambda_0\, uv$. In the 6×6 matrix form (16), the 3×3 constitutive matrices now read:

$$A^{ab} = -\,\lambda_0(1+2v^2)\delta^{ab}, \quad B_{ab} = \lambda_0(1-2u^2)\delta_{ab}, \quad C^a{}_b = D_b{}^a = -\,2\lambda_0 uv\delta^a_b. \tag{30}$$

As above, we calculate the TR-tensor density $\mathcal{G}$ and check this with the help of computer algebra. The corresponding Fresnel equation reads:

$$-\lambda_0^3(1+2v^2)\left[(1+2v^2)q_{\hat{0}}^2 - (1-2u^2)(q_{\hat{1}}^2 + q_{\hat{2}}^2 + q_{\hat{3}}^2)\right]^2 = 0\,. \tag{31}$$

Perhaps surprizingly we find a single effective light cone: no birefringence, just a different speed of light as compared to the case without torsion. Up to a conformal factor, the optical metric is obviously

$$
g_{\rm opt}^{\alpha\beta} = \begin{pmatrix} 1+2v^2 & 0 & 0 & 0 \\ 0 & -1+2u^2 & 0 & 0 \\ 0 & 0 & -1+2u^2 & 0 \\ 0 & 0 & 0 & -1+2u^2 \end{pmatrix}. \tag{32}
$$

This result (reduction of the quartic Fresnel surface to a unique light cone) implies that we can rewrite the constitutive tensor in the conventional form:

$$
\chi^{\alpha\beta\gamma\delta} = \varphi \sqrt{-g_{\rm opt}} \left(g_{\rm opt}^{\alpha\gamma} \, g_{\rm opt}^{\beta\delta} - g_{\rm opt}^{\alpha\delta} \, g_{\rm opt}^{\beta\gamma} \right) + \alpha \, \epsilon^{\alpha\beta\gamma\delta} \tag{33}
$$

with the effective dilaton field $\varphi = \lambda_0 \sqrt{(1+2v^2)(1-2u^2)}$.

According to the metric (32), photons would propagate isotropically but with a torsion-dependent speed. In astrophysical observations, this could show up in a certain deviation from the cosmological redshift predictions of general relativity theory. When combined with additional studies on the behavior of particles and fields with spin, such observations may provide an experimental proof for the existence of torsion.

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REFERENCES

[1] R.A. Puntigam, C. Lämmerzahl, and F.W. Hehl, *Maxwell's theory on a post-Riemannian spacetime and the equivalence principle*, Class. Quantum Grav. **14** (1997) 1347–1356. Eprint Archive: gr-qc/9607023

[2] M.P. Haugan and C. Lämmerzahl, *Principles of equivalence: their role in gravitation physics and experiments that test them.* In: *Gyros, Clocks, Interferometers...: Testing Relativistic Gravity in Space.* C. Lämmerzahl et al., eds. Lecture Notes in Physics Vol. **562** (Springer: Berlin, 2001) pp. 195–212. Eprint Archive: gr-qc/0103067.

[3] M. Haugan and C. Lämmerzahl, *On the experimental foundations of the Maxwell equations*, *Ann. Phys. (Leipzig)* **9** (2000) Spec. Issue, SI-119–124. Eprint Archive: gr-qc/0002075.

[4] F.W. Hehl and Yu.N. Obukhov, *How does the electromagnetic field couple to gravity, in particular to metric, nonmetricity, torsion, and curvature?* In: *Gyros, Clocks, Interferometers...: Testing Relativistic Gravity in Space.* C. Lämmerzahl et al., eds. Lecture Notes in Physics Vol. **562** (Springer: Berlin, 2001) pp. 479–504. Eprint Archive: gr-qc/0001010.

[5] Y. Itin, *Effective axion field induced by a torsion of spacetime*, to appear (2003).

[6] O. Preuß, *Astronomical tests of the Einstein equivalence principle.* Ph.D. Thesis, University of Bielefeld (November 2002).

[7] O. Preuss, S.K. Solanki, M.P. Haugan, and D.T. Wickramasinghe, *Neue Astronomische Tests des Einsteinschen Äquivalenzprinzips*, talk given by O. Preuss at the Annual Meeting of the German Physical Society (DPG) in Hannover (March 2003), see `http://www.dpg-tagungen.de/prog/html/gr_1.html#3`.

[8] R. Tresguerres, *Exact vacuum solutions of 4-dimensional metric-affine gauge theories of gravitation,* Z. Phys. **C65** (1995) 347–354.

[9] R. Tresguerres: *Exact static vacuum solution of four–dimensional metric–affine gravity with nontrivial torsion*, Phys. Lett. **A200** (1995) 405–410.

[10] F. Gronwald and F.W. Hehl, *On the gauge aspects of gravity*, in: *International School of Cosmology and Gravitation:* 14$^{\text{th}}$ Course: Quantum Gravity, held May 1995 in Erice, Italy. Proceedings. P.G. Bergmann et al. (eds.). World Scientific, Singapore (1996) pp. 148–198. Eprint Archive: gr-qc/9602013.

[11] M.P. Haugan and T.F. Kauffmann, *New test of the Einstein equivalence principle and the isotropy of space*, Phys. Rev. **D52** (1995) 3168–3175.

[12] M.D. Gabriel, M.P. Haugan, R.B. Mann, and J.H. Palmer, *Gravitational light deflection and propagation delay in nonsymmetric theories of gravity*, Phys. Rev. **D43** (1991) 308–313.

[13] F.W. Hehl and Yu.N. Obukhov, *Foundations of Classical Electrodynamics — Electric charge and magnetic flux conservation and the spacetime relation —*, Birkhäuser, Boston, MA (2003) in press.

[14] G.F. Rubilar, *Linear pre-metric electrodynamics and deduction of the lightcone*, Ann. *Phys. (Leipzig)* **11** (2002) 717–782.

[15] Yu.N. Obukhov and G.F. Rubilar, *Fresnel analysis of the wave propagation in nonlinear electrodynamics*, *Phys. Rev.* **D66** (2002) 024042 (11 pages). Eprint Archive: gr-qc/0204028.

[16] V. Perlick, *Ray Optics, Fermat's Principle, and Applications to General Relativity*, Lecture Notes in Physics, Monographs Vol. **m61**, 220 pages (Springer: Berlin, 2000).

[17] P. Baekler and F.W. Hehl: *A micro-deSitter spacetime with constant torsion: a new vacuum solution of the Poincaré gauge field theory*, in *Gauge Theory and Gravitation, Proceedings of the International Symposium on Gauge Theory and Gravitation,* Nara, Japan, August 1982, K. Kikkawa et al., eds., Lecture Notes in Physics (Spinger) Vol. **176**

(1983) pp. 1-15.

[18] H. Goenner and F. Müller-Hoissen, *Spatially homogeneous and isotropic spaces in theories of gravitation with torsion*, Class. Quantum Grav. **1** (1984) 651–672.

[19] A.V. Minkevich and A.S. Garkun, *Isotropic cosmology in metric-affine gauge theory of gravity,* Eprint Archive: gr-qc/9805007 (May 1998).